

# Short Course on Experimental Dynamic Substructuring

## Module #8: Transmission Simulator Method



THE UNIVERSITY  
of  
WISCONSIN  
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**Short Course Notes For:**

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## Component Mode Synthesis

- As discussed previously, if we have the modal parameters of two systems, either from analysis or experiment, we can write:

$$\begin{aligned} \begin{bmatrix} \mathbf{I}^A & \mathbf{0} \\ \mathbf{0} & \mathbf{I}^B \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}^A \\ \ddot{\mathbf{q}}^B \end{Bmatrix} + \begin{bmatrix} \backslash 2\zeta_r \omega_r \backslash^A & \mathbf{0} \\ \mathbf{0} & \backslash 2\zeta_r \omega_r \backslash^B \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}}^A \\ \dot{\mathbf{q}}^B \end{Bmatrix} + \begin{bmatrix} \backslash \omega_r^2 \backslash^A & \mathbf{0} \\ \mathbf{0} & \backslash \omega_r^2 \backslash^B \end{bmatrix} \begin{Bmatrix} \mathbf{q}^A \\ \mathbf{q}^B \end{Bmatrix} = \\ = \begin{bmatrix} (\Phi^A)^T & \mathbf{0} \\ \mathbf{0} & (\Phi^B)^T \end{bmatrix} \begin{Bmatrix} \mathbf{F}^A \\ \mathbf{F}^B \end{Bmatrix} \\ \begin{Bmatrix} \mathbf{y}^A \\ \mathbf{y}^B \end{Bmatrix} = \begin{bmatrix} \Phi^A & \mathbf{0} \\ \mathbf{0} & \Phi^B \end{bmatrix} \begin{Bmatrix} \mathbf{q}^A \\ \mathbf{q}^B \end{Bmatrix} \end{aligned}$$

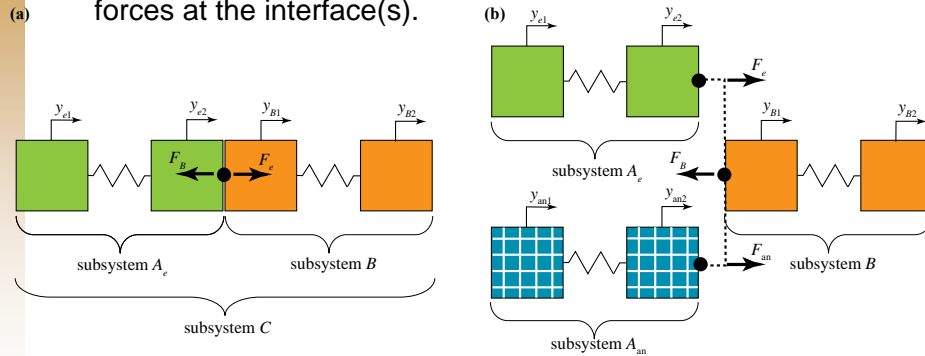
- Then, constraints in the following form are enforced to couple the substructures:

$$\mathbf{B}_p \begin{Bmatrix} \mathbf{y}^A \\ \mathbf{y}^B \end{Bmatrix} = \mathbf{0} \quad \mathbf{B} = \mathbf{B}_p \begin{bmatrix} \Phi^A & \mathbf{0} \\ \mathbf{0} & \Phi^B \end{bmatrix} \quad \mathbf{B} \begin{Bmatrix} \mathbf{q}^A \\ \mathbf{q}^B \end{Bmatrix} = \mathbf{0}$$



## Substructure Uncoupling

- Our approach to “uncouple” structures works by constraining a model of one component to the structure that cancels the forces at the interface(s).



$$\mathbf{I}_e \ddot{\mathbf{q}}_e + \left[ \begin{smallmatrix} 2\zeta_r \omega_{r\backslash} \end{smallmatrix} \right]_e \dot{\mathbf{q}}_e + \left[ \begin{smallmatrix} \omega_{r\backslash}^2 \end{smallmatrix} \right]_e \mathbf{q}_e = \Phi_e^T \mathbf{F}_e$$

$$+ \mathbf{I}_{an} \ddot{\mathbf{q}}_{an} + \left[ \begin{smallmatrix} 2\zeta_r \omega_{r\backslash} \end{smallmatrix} \right]_{an} \dot{\mathbf{q}}_{an} + \left[ \begin{smallmatrix} \omega_{r\backslash}^2 \end{smallmatrix} \right]_{an} \mathbf{q}_{an} = \Phi_{an}^T \mathbf{F}_{an}$$

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## The interfaces forces can be made to cancel if:

- 1.) The modal parameters of the analytical and true fixtures are equal and opposite, i.e.  $\left[ \begin{smallmatrix} \omega_{r\backslash}^2 \end{smallmatrix} \right]_{an} = -\left[ \begin{smallmatrix} \omega_{r\backslash}^2 \end{smallmatrix} \right]_e$  and similarly for the mass and damping terms.
- 2.) The motion of both the experimental and analytical fixtures is the same,  $\{\mathbf{q}_{an}(t)\} = \{\mathbf{q}_e(t)\}$
- 3.) The mode shapes of both the experimental and analytical models are equal  $\Phi_{an} = \Phi_e$ .
- 4.) The interface force vector is in the range space of  $\Phi_{an}^T$ .

$$\mathbf{I}_e \ddot{\mathbf{q}}_e + \mathbf{I}_{an} \ddot{\mathbf{q}}_{an} + \left[ \begin{smallmatrix} 2\zeta_r \omega_{r\backslash} \end{smallmatrix} \right]_e \dot{\mathbf{q}}_e + \left[ \begin{smallmatrix} 2\zeta_r \omega_{r\backslash} \end{smallmatrix} \right]_{an} \dot{\mathbf{q}}_{an} + \left[ \begin{smallmatrix} \omega_{r\backslash}^2 \end{smallmatrix} \right]_e \mathbf{q}_e + \left[ \begin{smallmatrix} \omega_{r\backslash}^2 \end{smallmatrix} \right]_{an} \mathbf{q}_{an} =$$

$$= \Phi_e^T \mathbf{F}_e + \Phi_{an}^T \mathbf{F}_{an}$$

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## Connection Methods – CPT and MCFS

- One cannot measure the response at the connection point directly, so it must be estimated from other measurements.
  - CPT Method: Connection point responses for the experimental system are estimated using a modal filter and constrained to the analytical fixture A and beam D:

$$\begin{Bmatrix} \{y^C\}_m \\ \{y^C\}_c \end{Bmatrix} \approx \begin{bmatrix} [\Phi_m^A] \\ [\Phi_c^A] \end{bmatrix} \{q^C\} \rightarrow \{y^C\}_c = [\Phi_c^A] [\Phi_m^A]^\dagger \{y^C\}_m$$

- Modal Constraint Method: Constrain the modal DOF of the Fixture model to their approximation on C: (aka MCFS or “Modal Constraint for Fixture and Subsystem”)

$$\begin{cases} \{q^A\} = [\Phi_m^A]^\dagger \{y^A\}_m \\ \{q^C\} = [\Phi_m^A]^\dagger \{y^C\}_m \end{cases} \rightarrow \{q^A\} = \{q^C\}$$

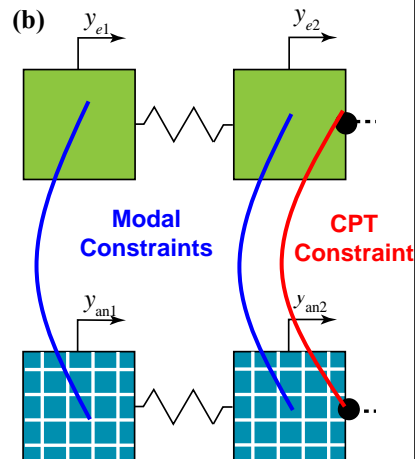


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## Connection Methods – Rationalization

- CMS can be very sensitive to errors when removing a substructure from a system.
- If the fixture is elastic, then it will probably have more modal DOF ( $N_A$ ) than there are connection point DOF ( $N_C$ ).
  - e.g. requiring equal 2D motion at the connection point enforces only (3) constraints.
  - When the number of constraints is  $N_C < N_A$ , it is unlikely that the modes of both structures will match and the interface forces most likely will not cancel!

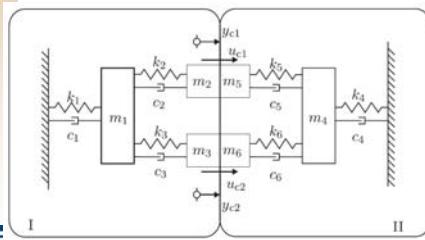


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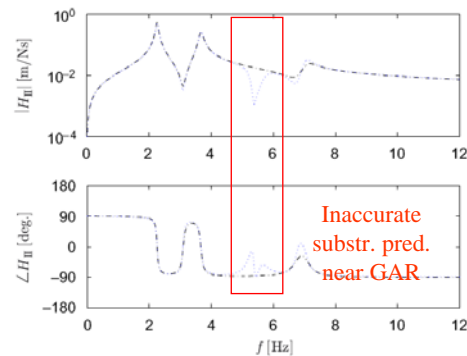
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## Related Techniques

- Sjovall & Abrahamsson observed a similar phenomenon using FBS and called it “generalized anti-resonance” (GAR).
- Suggested using internal DOF to eliminate motions that are poorly observable on the interface DOF.
  - *W. D'Ambrogio and A. Fregolent, (IMAC XXVII) Orlando, Florida, 2009.*
  - *Sjovall, P. and T. Abrahamsson (2008). MSSP 22(1): 15-33.*



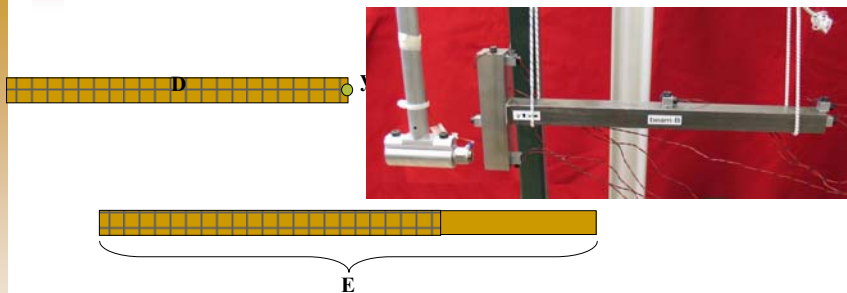
$$H(\omega_{\text{GAR}})U = 0 \text{ for } U \neq 0$$



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## Test Case #1: T-Beam

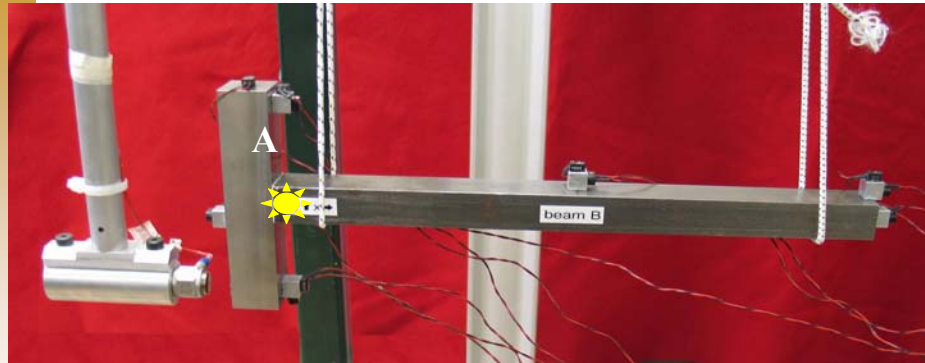


- Objective: Join an experimental model of beam B to an analytical model for beam D at point  $y_c$ .
  - Measurements on system C = beam B + fixture A
  - Analytical model (Euler-Bernoulli) for fixture A removed from C
  - Beam B + Beam D (tuned Euler-Bernoulli analytical model) = assembly Beam E

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## Experimental Procedure



- Careful tests used to estimate the modal parameters of the C system at a number of points on fixture A.
- Hammer used to excite the largest bandwidth (largest number of modes) possible and to avoid modifying structure with the exciter.

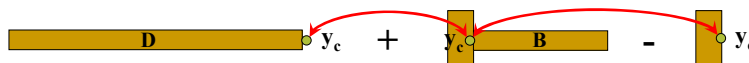


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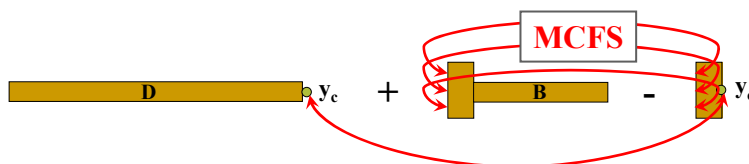
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## Cases Considered

- Case 1: **CPT**: Models for A, C and D joined at the connection point.
  - Case 1a: Rigid Fixture Model
  - Case 1b: Elastic Fixture Model



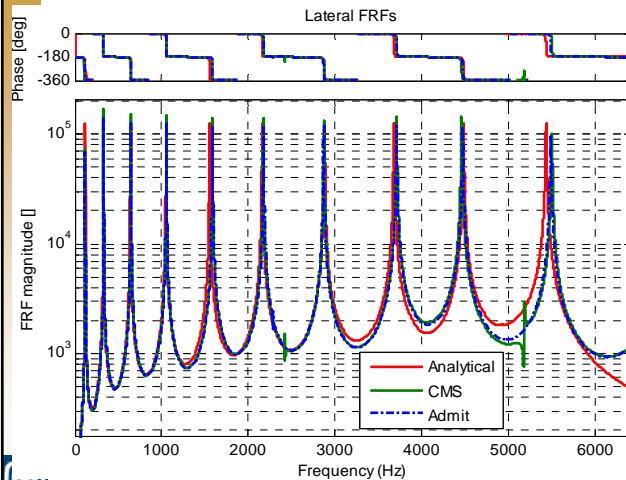
- Case 2: **MCFS**: Models for A and C joined using MCFS method.
  - Elastic Fixture Model



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## Case 1a: Rigid A, CPT



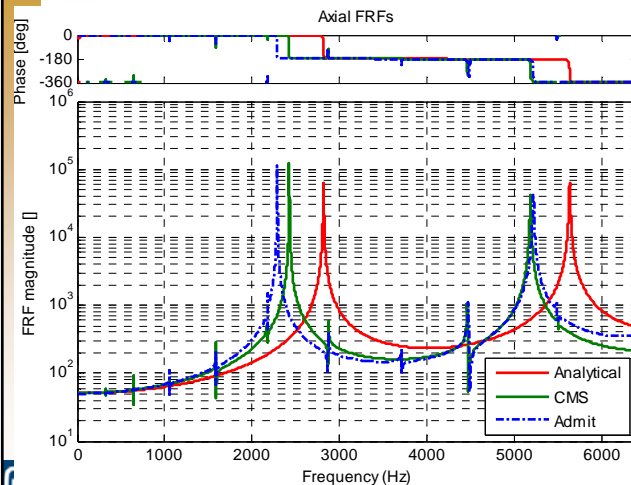
“Admit” = FBS with  
reconstructed FRFs

- Excellent results obtained in the lateral direction (Bending Modes).
- Both the CMS and FRF based Admittance procedures agree very well with the analytical model.
- The CMS result is slightly contaminated by the axial modes at 2400 and 5200 Hz.

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## Case 1a: Rigid A, CPT

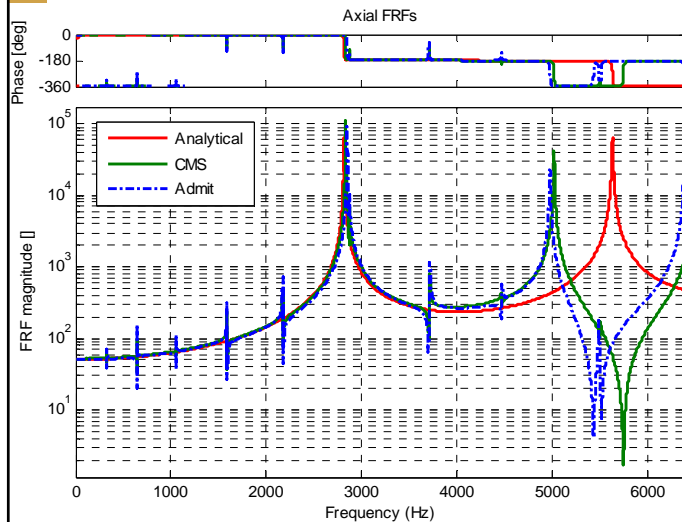


- Admittance and CMS both under-predict the natural frequencies of the axial modes by 10% or more.
  - Fixture flexibility is important!
- Contamination at the bending natural frequencies.
  - Possibly due to small curve fitting errors or cross axis sensitivity.

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## Case 1b: Flexible A, CPT



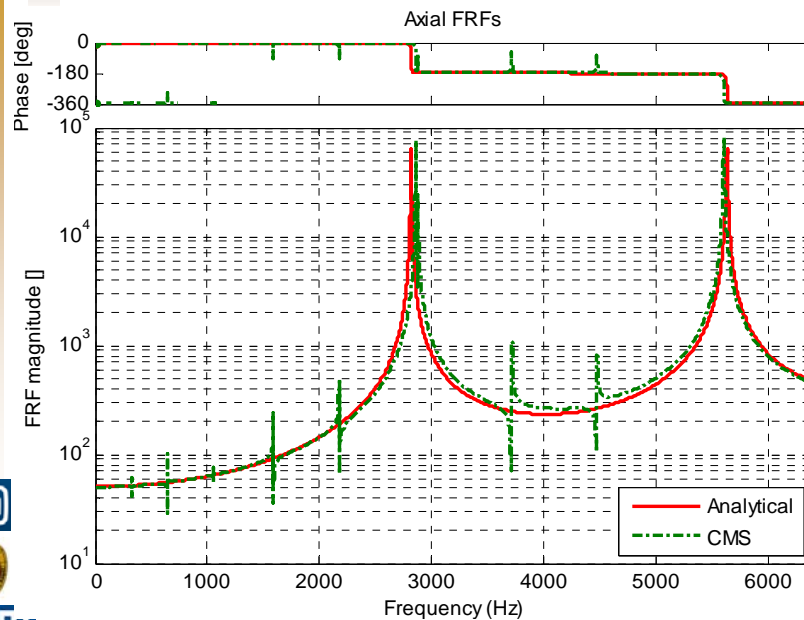
- Both FBS and CMS accurately predict the first axial frequency with the flexible fixture model.
- Both methods more severely under-predict the second axial mode.
- Both predict a spurious zero near 5500 Hz.
- The Lateral FRFs were similar to those shown previously.



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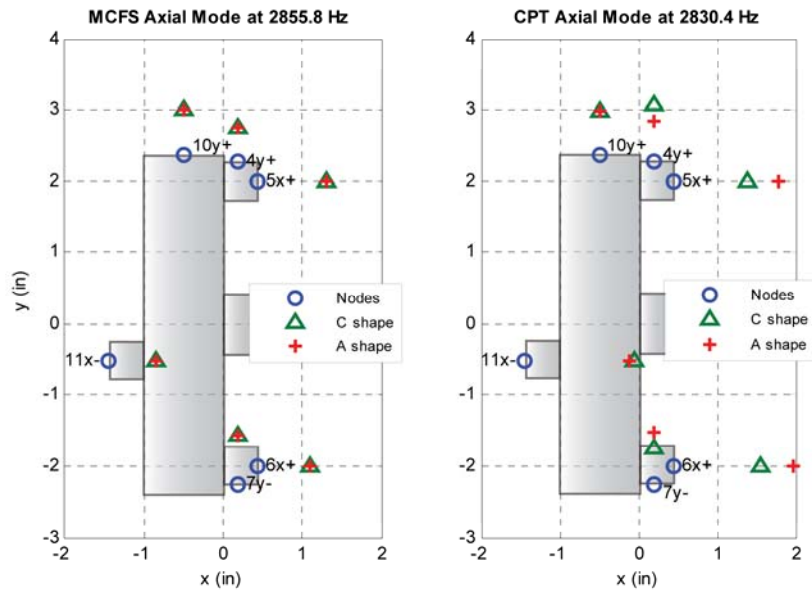
## Case 2: Flexible A, MCFS Method



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## Explanation:

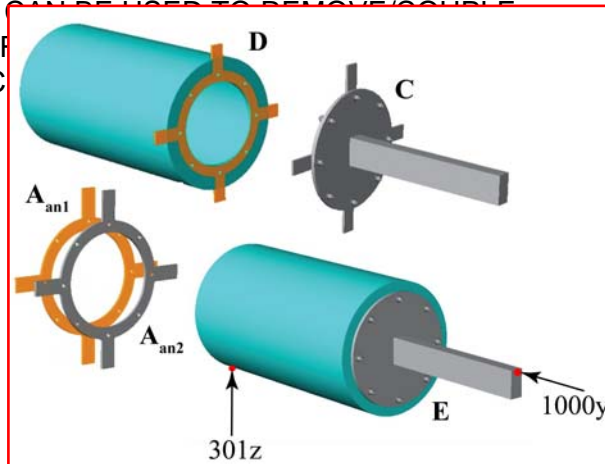


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## Significance:

- Now it is possible to robustly remove one flexible structure from another.
- The fixture modal basis gives a convenient basis for the interface – CAN BE USED TO REMOVE/Couple



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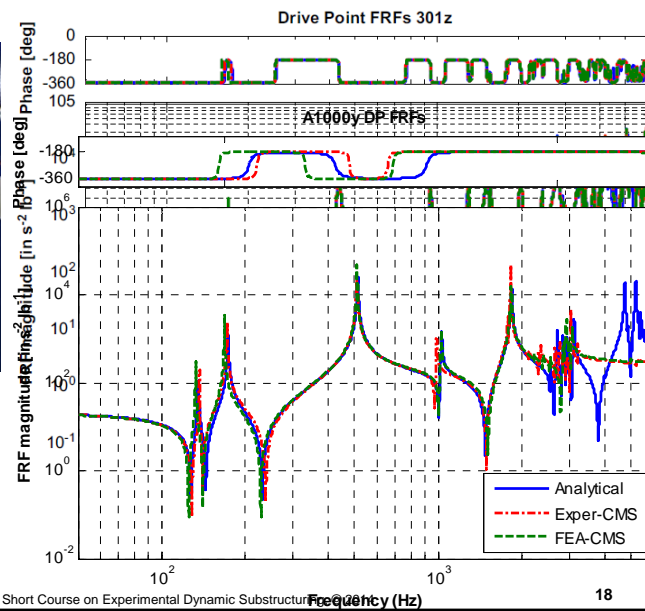
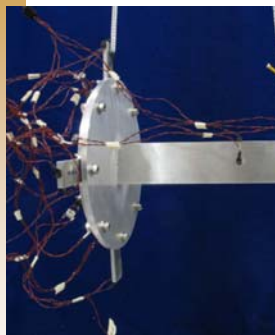
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## Transmission Simulator Method Enables use of Continuous Interfaces



## Transmission Simulator Method Enables use of Continuous Interfaces



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## Transmission Simulator Design

- Design the TS so that it can be instrumented to capture all of its relevant dynamic modes so that  $[\Phi_m]$  will be well conditioned for inversion.
- Choose the mass of the fixture to bring more modes into the testable bandwidth.
- Avoid designs that contain joints, intricate geometry, or other features that are difficult to accurately model analytically. Making the fixture of one piece, without any joints, goes a long way toward meeting this guideline.
- The joint between the fixture and the test article should replicate the actual joint in the system of interest as closely as possible, to capture the joint stiffness and damping.
- The fixture's impedance should roughly resemble that of the built-up structure. The fixture might just be a chopped off version of the other substructure which precisely mimics the mass and stiffness near the connection interface.



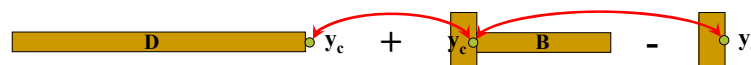
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## Problem of Negative Mass (& Stiffness)

- The transmission simulator must be removed to obtain the desired experimental model.
- The subtraction process may introduce negative mass and stiffness.
- Models with negative mass and stiffness are not readily imported into FEA packages, and they may introduce errors or counter-intuitive results.
- Example: Beam problem mentioned earlier

	Complex $f_n$ (Hz)	$\text{eig}(M) < 0$
<b>Case 1a</b> Rigid Fixture, Con. Pt.	$0 + i*8.9e-5$ $0 + i*5.5e5$	-0.011
<b>Case 1b</b> Flexible Fixture, Con. Pt.	$0 + i*1.36e-4$ $8951 - i*2450$ $8951 + i*2450$	-1
<b>Case 2</b> Flexible Fixture, MCFS	$0 + i*2.28e-4$ $13050 - i*4285$ $13050 + i*4285$	-0.086



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## Modal Scale Factor Method (Mayes)

- Negative mass implies that too much mass is being subtracted → the scale factors of the transmission simulator modes may be in error.

□ Solution: Adjust the modal scale factors to produce a positive-definite mass matrix. But how?

- Eigenvalues of Mass Matrix of B (structure of interest)

$$\mathbf{M}_B \boldsymbol{\psi}_k = \lambda_k \boldsymbol{\psi}_k \quad \lambda_k < 0 \quad k = 0 \dots N_{\text{neg}}$$

- There are typically only a few negative eigenvalues and several modal scale factors ( $\text{mm}_j$ ) that could be modified → under-constrained optimization problem.

$$\mathbf{f}(\text{mm}_j) = [\lambda_k \quad \dots \quad \lambda_{N_{\text{neg}}}]^T$$

$$\mathbf{f}(\text{mm}_j + \Delta \text{mm}_j) = \mathbf{f}(\text{mm}_j) + [\nabla \mathbf{f}(\text{mm}_j)]_{\text{mm}_j} \Delta \text{mm}_j$$



## Added Mass Method (Allen)

- Negative mass can be eliminated by adding mass to the structure until the total mass is positive.

□ Suppose point masses are added at any of the points where the system's modes were measured. The resulting EOM are identical only with the mass matrix replaced by:

$$\mathbf{M}_B + \Delta \mathbf{m}$$

□ The smallest change to the mass matrix that will make all of its eigenvalues positive is:

$$\Delta \lambda_k = \begin{cases} 0 & \lambda_k > 0 \\ -\lambda_k + \varepsilon & \lambda_k \leq 0 \end{cases} \quad \mathbf{M}_B \boldsymbol{\psi}_k = \lambda_k \boldsymbol{\psi}_k$$

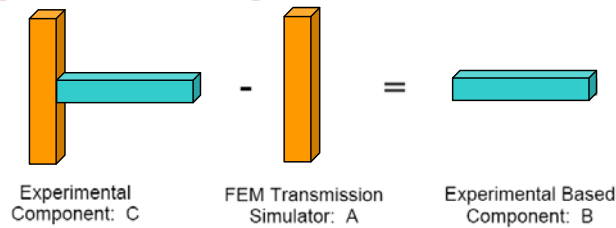
□ where  $\varepsilon$  is a small value. This can be obtained using:

$$\Delta \mathbf{m} = \boldsymbol{\psi} \hat{\Lambda} \boldsymbol{\psi}^T \quad \boldsymbol{\psi} = [\boldsymbol{\psi}_k \quad \dots \quad \boldsymbol{\psi}_{N_c}]$$

$$\hat{\Lambda} = \text{diag}[(\lambda_1 + \Delta \lambda_1) \quad \dots \quad (\lambda_{N_c} + \Delta \lambda_{N_c})]$$



## Application Examples



- Modal test obtains all modes below 20kHz for C and below 24kHz for A → 7 modes for TS and 15 modes for C
- The resulting model for B has two negative eigenvalues:
  - $\lambda_1 = -0.00050468$
  - $\lambda_2 = -2.4058e-16$ , (essentially zero).
  - We desire to make these eigenvalues positive using the proposed methods.

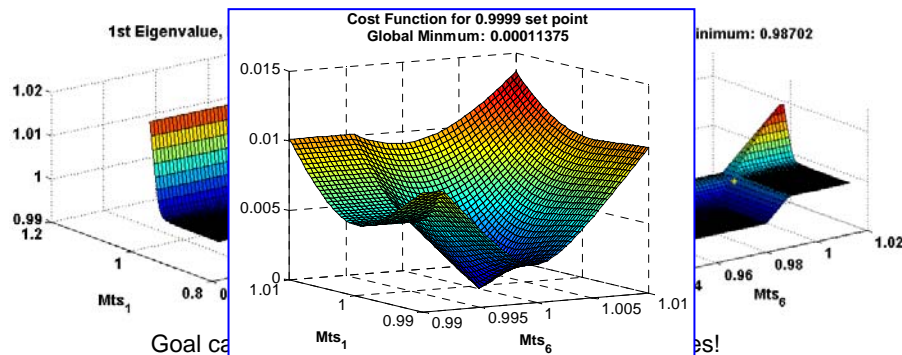


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## Results: Modal Scale Factor Method

- Jacobian (or metrics) reveal that the negative eigenvalues are most strongly affected by modes 1 and 6.
- Using  $\varepsilon=0.9999$ , after 3 iterations the algorithm converges on:
  - $M_{ts,1}=0.99824$  and  $M_{ts,6}=0.9999$
  - (or increase mode scale factors by: 1.0008798 and 1.000050)
- Works perfectly, however, what if we chose  $\varepsilon=0.99$ ?

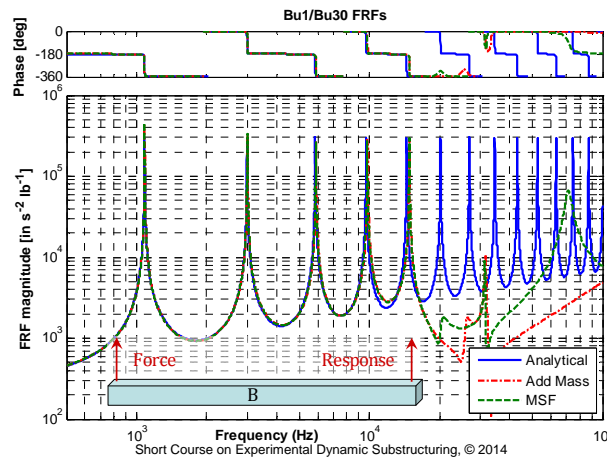


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## Results: Added Mass Method

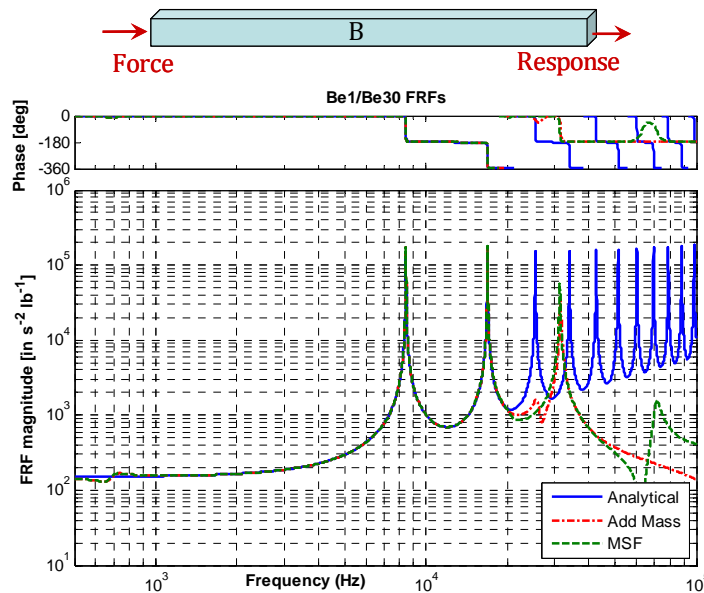
- Using  $\varepsilon=2.2\text{e-}14$  (100 times larger than machine precision), smallest eigenvalue becomes  $2.2\text{e-}14$  as expected.
- How much mass had to be added to achieve this?
  - Ratio of the norm of  $\mathbf{M}_B$  to  $\Delta\mathbf{m} = 0.000505$  (less than 0.05%!)



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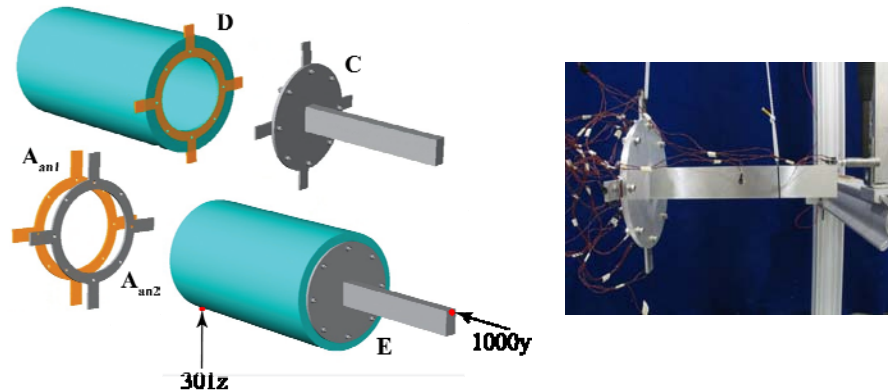
## Axial FRFs: Both Methods



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## Three-Dimensional Plate-Beam System



- Test performed to measure the modes of C
- Analytical system D modeled with finite elements with a second copy of the transmission simulator
- C & D are joined and then two copies of the transmission simulator are removed (using the FEA model of the TS)

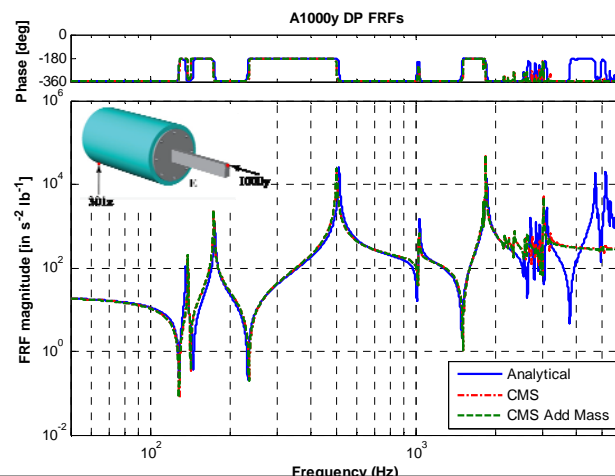


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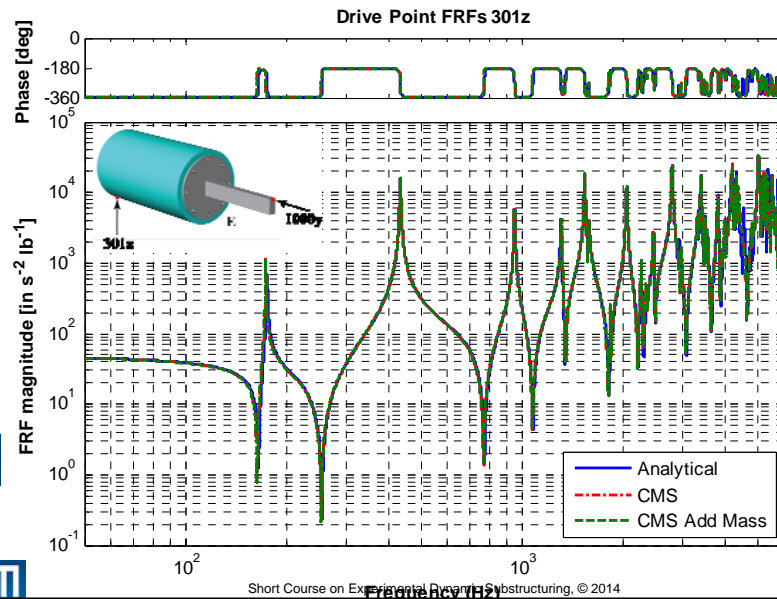
## Case 1: Correct Mass Matrix after Assembling Full System, C+D-2A

- 4 Negative Eig.:  $\lambda_1 = -0.197$ ,  $\lambda_2 = -0.0764$ ,  $\lambda_3 = -0.134$ ,  $\lambda_4 = -0.118$
- Used added mass method, which revealed that 19.7% more mass had to be added to make the mass matrix positive definite.
- No deterioration observed in the quality of the FRFs



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## Case 1: Correct Mass Matrix after Assembling Full System, C+D-2A

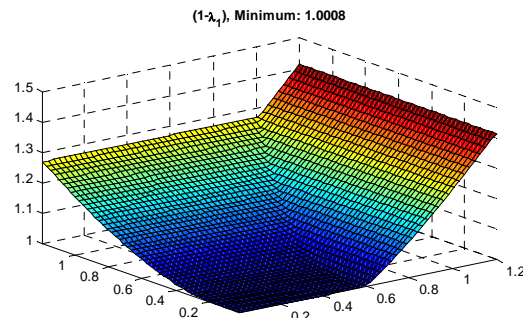


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## Case 2: Correct the mass matrix for C-A

- After removing the transmission simulator from C, the resulting model for B has two negative eigenvalues:
  - $\lambda_1 = -0.116$  and  $\lambda_2 = -0.0865$
  - Using added mass method, the norm of the required mass addition is 11.6% of the norm of  $M_C$
  - Mode Scale Method implemented with the Nelder-Mead Simplex algorithm using the dominant modes found by Kammer's  $\tau$  metrics, the 9<sup>th</sup> and 13<sup>th</sup> modes.
    - Algorithm produced a negative modal mass, which was clearly not reasonable.



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Mts 9

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## Conclusions

- Two methods have been proposed for correcting the negative mass that is sometimes obtained in substructuring predictions.
- One is highly unlikely to find a set of scale factors that produce positive mass using trial and error, so these new algorithms are a significant aid!
- Added Mass Method
  - Is straightforward to implement and was found to be successful in a variety of challenging cases.
  - Sometimes quite a large amount of mass had to be added.
- Modal Scale Factor Method:
  - This approach is more easily justified physically, since it avoids subtracting too much mass rather than correcting negative mass after the fact.
  - A nonlinear optimization problem must be solved to compute the modal scale factors of the transmission simulator that produce the desired result.



## Additional Comments by R.L. Mayes

